

Grade Level/Course: Grade 7 Mathematics
Lesson/Unit Plan Name: Gears and Gear Ratios
Rationale/Lesson Abstract: This lesson applies fractions and ratios to an investigation and construction of gears and gear trains. In a Fab Lab, students can also produce gears using a laser cutter and gear cars using a 3D printer, and analyze the gear ratios as an assessment.
Timeframe: 1 week
Common Core Standard(s): 7.RP.3 Additionally: 7.RP.1, 7.RP.2, 7.EE.3, 7.EE.4, 7.G.2, 7.G.4

Instructional Resources/Materials:

Paper, pencils, and markers
AutoCad or equivalent software to design the gears
Laser cutter to make the gears
3D printer to print the parts for the gear car (optional)

Activity/Lesson:

In this lesson students will use what they already know about fractions to investigate how gears work.

Explain to students:

Imagine you are trying to design a bike gearbox. First you will make a set of gears to learn the relationship between gear size, number of teeth, torque and speed. Then you will be able to make a more complicated gearbox that uses more than one gear to significantly increase or decrease the torque of a motor.

First, we will need to talk about how gears work.

Gears are a way to exchange speed and torque. You probably already know what speed is. In the case of gears, we are going to talk about how quickly a gear or wheel rotates. This is called **rotational speed**. Rotational speed is measured in **rpm** or **rotations per minute**. A gear that rotates at 5 rpm, for instance, makes five full rotations each minute.

Torque is like force, but for rotation. If you try biking up a hill but you can't make it up because it's too steep, you don't have enough torque to make it up. There is a tradeoff between **rotational speed** and **torque**, so that when you select a lower gear, you are lowering the speed and increasing the torque.

Gears are like wheels that are connected to each other by **teeth**. The **teeth** are the little nubs on the end of the gears that let the gears interlock and transfer power. If gears didn't have teeth, they would slip past each other. Gears are often different sizes, but in order for them to work with each other, they need to have teeth that are the same size.

Gears aren't any good by themselves. You need more than one gear to be combined together in order to change the speed and torque. A set of interlocking gears is called a **gear train**. When you have a **gear train**, the ratio of speeds between the gears is called the **gear ratio**.

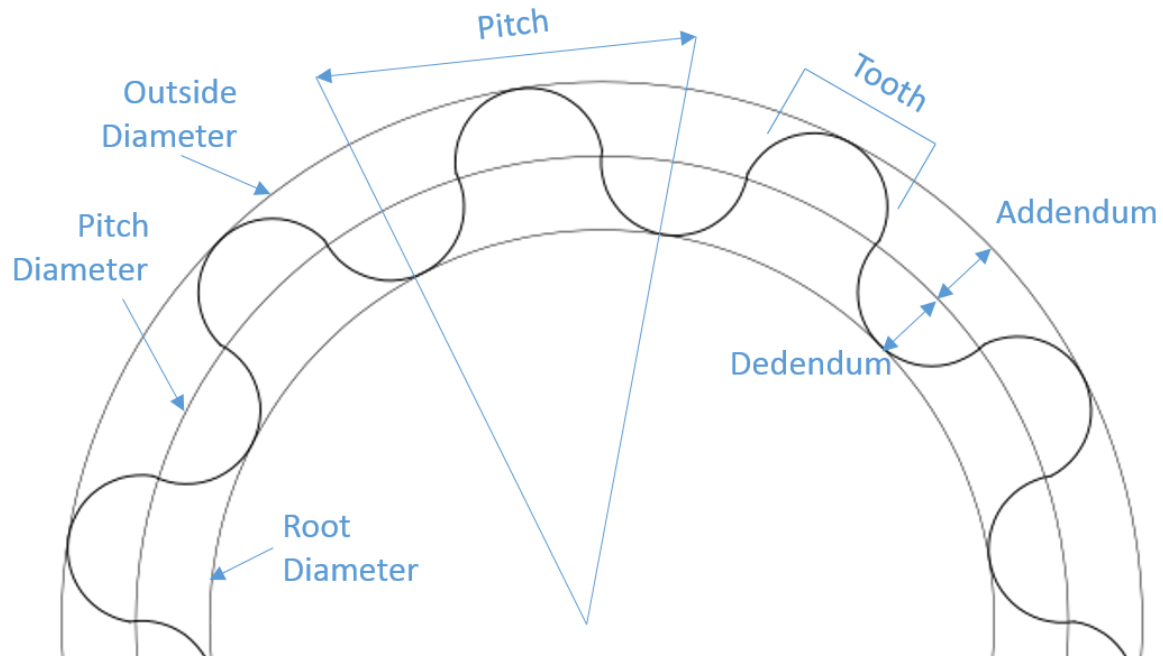
So let's say we have a gear train, and it consists of two gears, gear A and gear B. Gears can be compared to each other by their gear ratios. The gear ratio is R , and it describes the ratios of the angular velocities (ω), the radii of the two gears (r), or the number of teeth (N).

$$R = \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{N_B}{N_A}$$

Gear Construction

We've talked about the gear ratios, which we need for designing gear chains. But, we need to do some work to design the gear teeth themselves. Designing gear teeth can get really complicated if you want to make one for a machine. The reason is that the teeth need to be able to make contact without getting stuck. There are lots of shapes that can do it, but we're going to use semi-circles.

The shape of the gears we are going to make is shown in the picture below.



You can make the gears using AutoCAD. First, you need to decide what size you want for the tooth. Remember that it should be small enough so that you can make gears. About a quarter of an inch is a good size to start with.

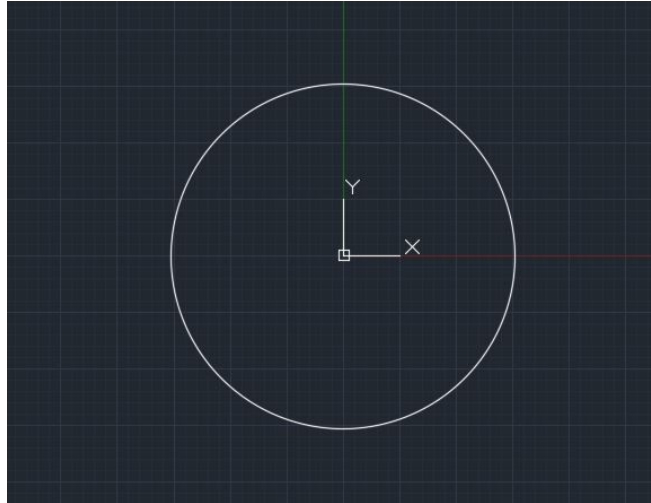
Now, let's say you want to make a gear with 10 teeth. You first need to make a circle that will be the **pitch circle** of your gear. The diameter of this circle is called the **pitch diameter**, which we can use to calculate the gear ratio like before. If you have 10 teeth, and a quarter-inch tooth, your effective circumference will be:

$$C_A = 2 \times N_A \times d_T$$

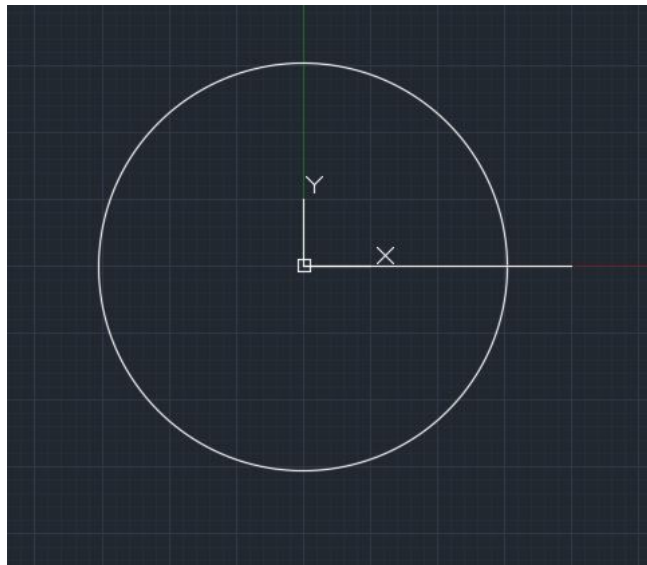
Where N_A is the number of teeth you want in your gear, and d_T is the width (or diameter) of a tooth. You need the 2 because each tooth will need both a ridge (the **addendum**) and a trough (the **dedendum**). This means that your numerator should be roughly the circumference of your gear. If you divide this by 2π , you will get the effective radius of the gear, like so:

$$r_A = \frac{C_A}{2\pi}$$

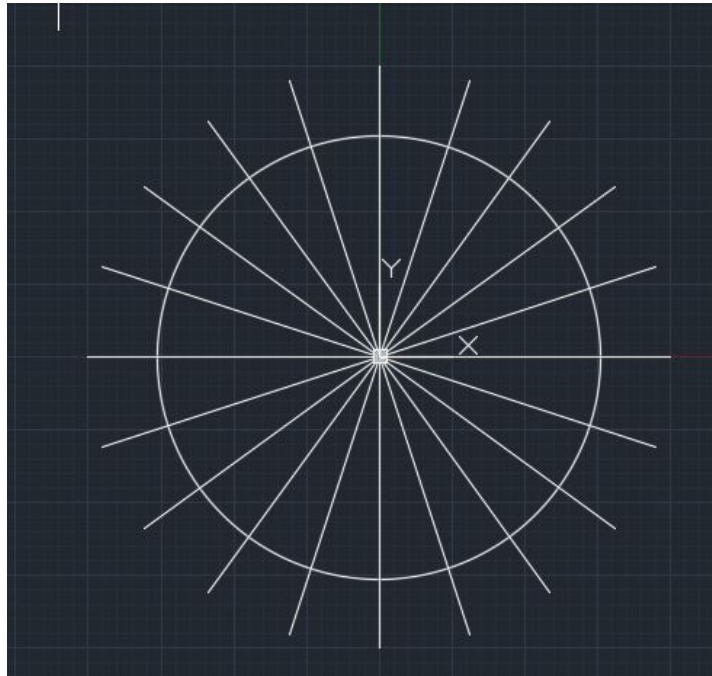
Once you have this number, you can start constructing your gears! You can do this in AutoCAD, so that once you are finished, you can send your designs on to the laser cutter to be made into real gears. Start by making a circle with the effective radius that you found before.



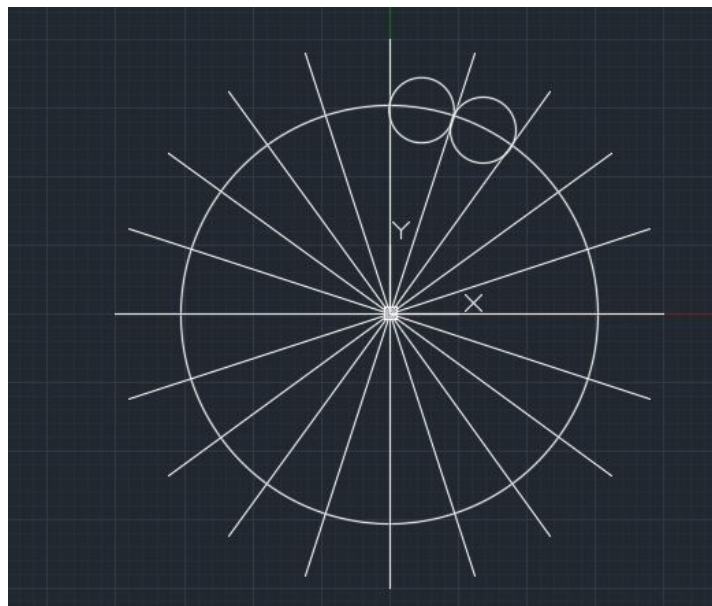
Now draw a single line going up on the x-axis. It doesn't really matter how high it goes, as long as it is longer than the circle.



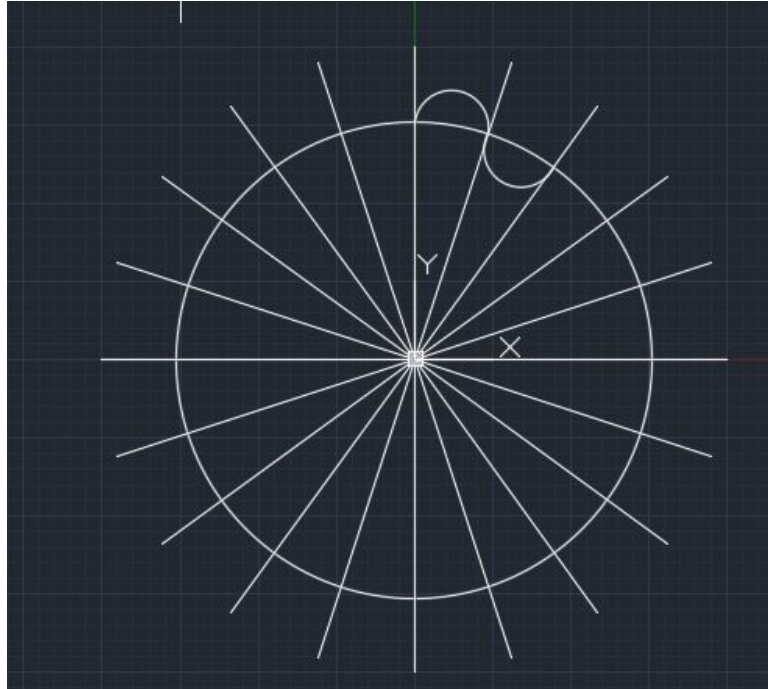
Now, you can create a polar array of lines, which will look like a pizza when you are all done. Click on the triangle to the right or the "Array" button in AutoCAD, and pick "polar array". Then, click on your line and press "Enter". Then AutoCAD will ask you to pick the center point of your array. Pick the middle of the circle, and press "Enter". You will see a pattern rotated around your circle so that it looks like a pizza or a sun. Look up at the top-left corner of your screen, and you will see that your parameter called "Items" is set to 6. Change it, so it is twice the number of teeth that you want (let's say 10, for a 5 tooth gear). You should get this shape:



Now it's time to make the teeth! Click on the triangle below the "Circle" box and pick the option that says "2-Point". This lets you make a circle by picking two points. Pick one point at the intersection of the vertical line and the big circle, and the other at the next intersection going clockwise. Now make another circle the same way, going clockwise. You should get something that looks like this:

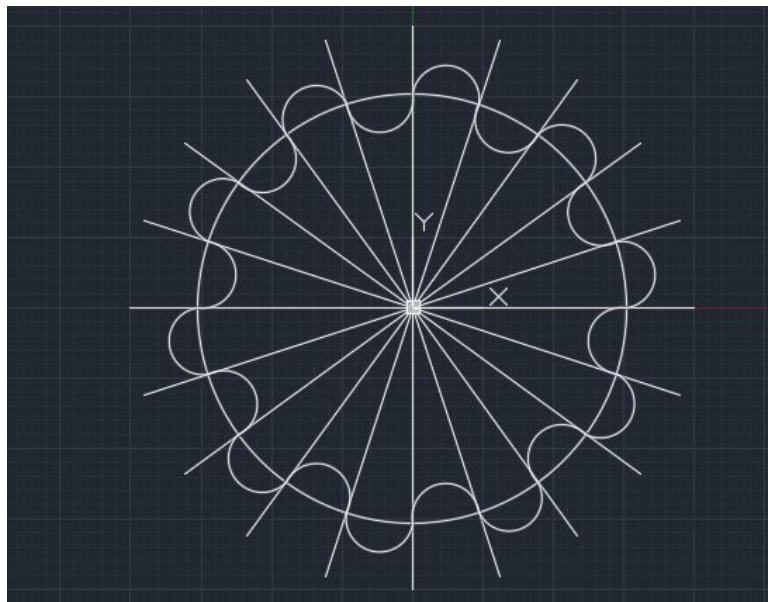


Now you are going to get rid of the extra parts of these circles that you don't need. Click on the button that says "Trim", then click on the big circle and press "Enter". Now click the bottom half of your first circle, and the top half of your second circle. They should each disappear when you do this. You will get this shape when you are done:

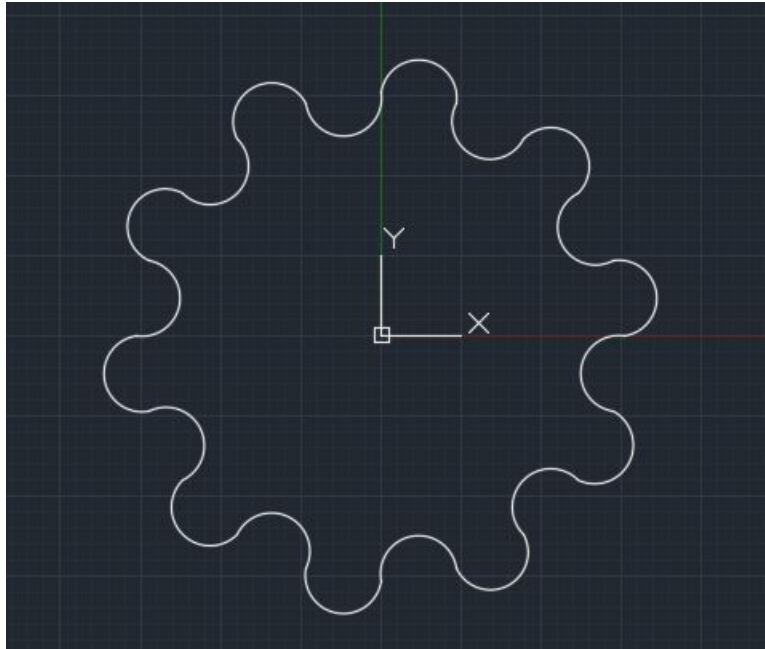


The semi-circle that makes the top of the tooth is called the **addendum**, while the part of the tooth on the bottom is the **dedendum**.

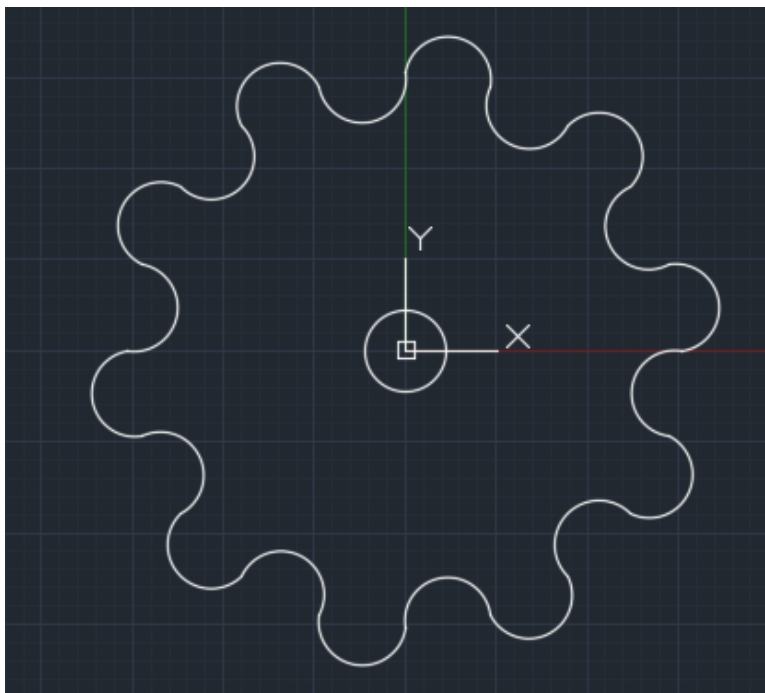
Now, you need to pattern your gear the whole way around, using the “Polar Array” command you did before. This time, select both semi-circles, and choose the center of your big circle as the center. Then use the “Polar Array” command to get the teeth to pattern the whole way around the gear.



Once you’ve finished. You can delete your circle and lines. This will leave you with just the gear.



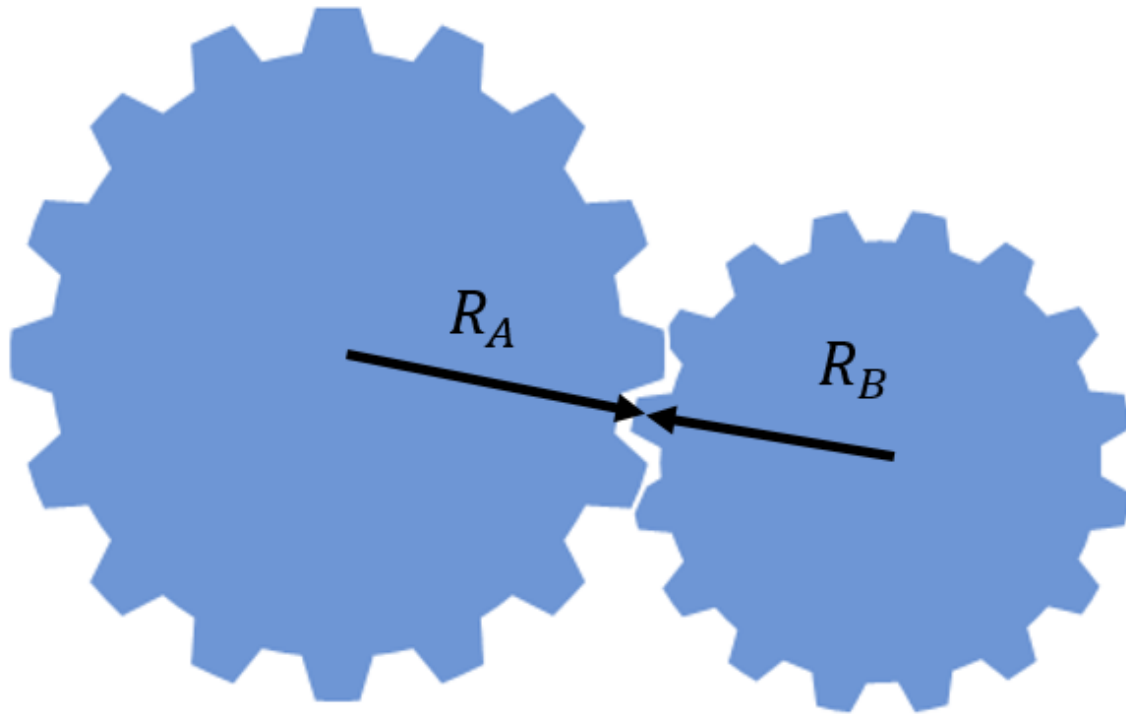
Finally, make sure to put a circle in the middle of the gear. This will be the axle hole, which the gear spins around.



When your gear design is done, you can export it to the gear layout to the laser cutter. You can make gears out of wood or acrylic (plastic).

Connecting Gears Together

Now that you can make a gear, it is time to construct a **gear train**. Let's first talk about a simple gear train with two gears. Make one gear three times larger than the other, and connect them together. Gears spin on an **axle**, and the axles must be laid out so that the gears are the perfect distance apart. The distance between any two gears is the sum of the pitch radii.

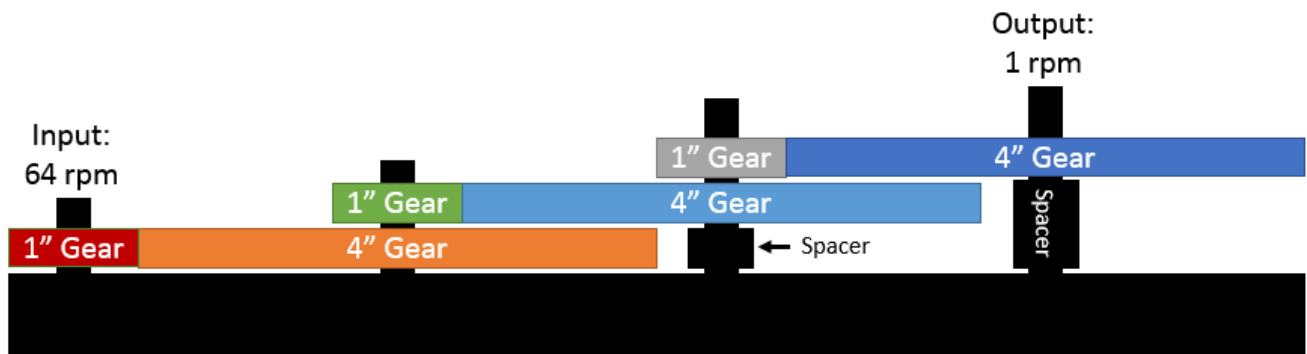


Once have set up the gears, use a red marker to mark one of the teeth on each.

- Take the smaller gear and slowly rotate it one full rotation. How far did the bigger gear go?
- Now rotate the bigger gear one full rotation. How far did the smaller gear rotate?

Constructing Longer Gear Trains

Now it's time to try making longer gear trains. Longer gear trains are made with **compound gears**. A compound gear is made of two or more gears that are attached to each other. You can use glue to connect your two gears to each other. If you want a stronger connection, you can bolt your gears together.



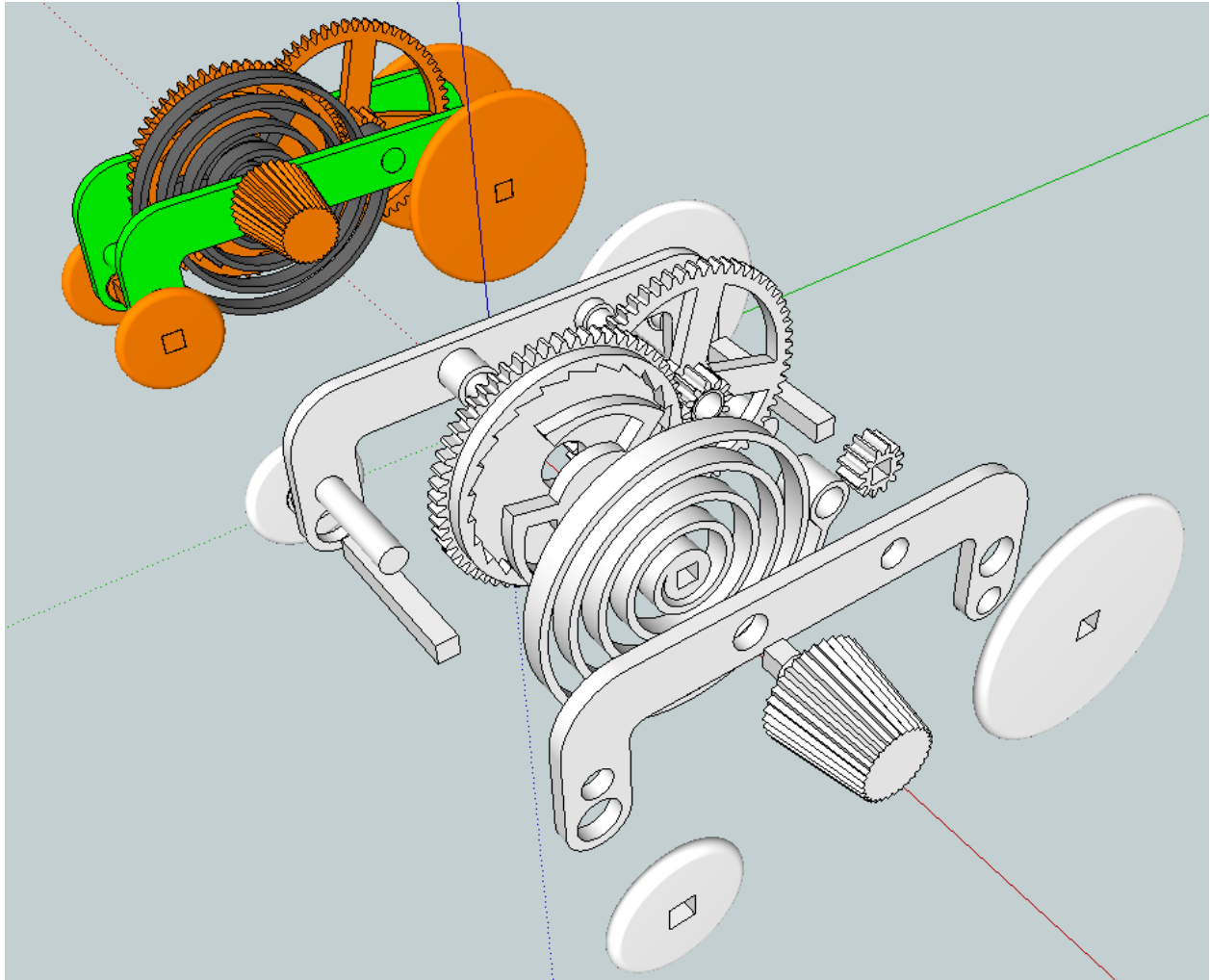
This diagram shows a sample compound gear train. This gearbox takes a gear that spins at 64rpm, and goes down to 1rpm. The way we can figure it out is:

$$64\text{rpm} \times \frac{1"}{4"} \times \frac{1"}{4"} \times \frac{1"}{4"} = 1\text{rpm}$$

Now it's your turn! Make a compound gear train that increases the speed of the input gear by 320 times using three compound gears.

Making a Spring-Powered Car

Now that you have a working knowledge of gears, let's see how they fit together into a real working system. Below, you can see a picture of a spring-powered car.



All of the parts are shown separately in white, and connected together in color. You can print all the parts out of plastic using the 3D printer. The parts probably won't fit together right away. You'll need to clean up the parts with sandpaper and a blade if they don't fit. Once the parts fit, add dry silicone lubricant to any of the moving parts.

Assessment:

Once you put the parts of the gear car together, take a look at how the car works.

What is the gear ratio?

If you turn the knob one full rotation, how far will the wheels turn?

The easiest way to measure the gear ratio is to count the number of teeth on the gears and use the formula on the first page. Once you know the gear ratio, take a minute to think about why the car has gears in the first place.

First, how many full rotations can you turn the knob before the spring is fully tightened?

Now how many rotations does the wheel make when you let the car go? Hint: measure how far the car travels with a tape measure. Then remember that the circumference of the wheel is $\pi \times d$. So the number of rotations that the wheel makes is distance that the car travels, divided by the wheel circumference, or

$$\# \text{ of rotations} = \frac{\text{distance traveled}}{\pi \times (\text{wheel diameter})}$$

Using this equation, the car makes _____ rotations.

Now divide the number rotations that the wheel makes by the number of rotations that the knob makes. How does this compare to the gear ratio?

Finally, if the gear ratio is too low, the car wouldn't travel as far. Can you think of why the gear ratio isn't any higher?

What would happen if the gear ratio was too high?